

# Isomerism and Vibration Spectra of $M_2XO_4$ Molecules ( $M = \text{Li, Na, K}$ ; $X = \text{S, Se, Te, Cr, Mo, W}$ )

V. G. Solomonik and A. V. Marenich

Ivanovo State University of Chemical Engineering, Ivanovo, Russia

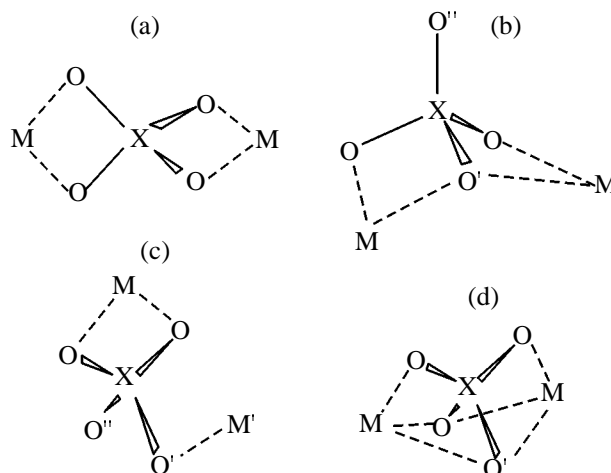
Received June 19, 2000

**Abstract**—The equilibrium geometries, isomerization energies, force fields, vibration frequencies, and band intensities in the IR spectra of  $M_2XO_4$  molecules ( $M = \text{Li, Na, K}$ ;  $X = \text{S, Se, Te, Cr, Mo, W}$ ) were calculated *ab initio* by the Hartree–Fock method in extended basis sets using relativistic effective core potentials. The relative energies of alternative structures were refined by the configuration interaction method taking into account single- and double-excited configurations, with the Davidson correction for quartic excitations. The results show that the chemical bonds between the metal atom and the acid residue  $XO_4$  are highly polar. The majority of  $M_2XO_4$  molecules have two isomers. In both isomers the  $XO_4^{2-}$  anion coordinates the metal cations  $M^+$  in the bisbidentate (*bb*) fashion. The equilibrium configurations of the nuclei in the ground (*bb*) and excited (*bb'*) isomers have the  $D_{2d}$  and  $C_s$  symmetry, respectively. In the *bb* isomer, the cations coordinate at the opposite, and in the *bb'* isomer, at the adjacent edges of the  $XO_4^{2-}$  anion, having the shape of a distorted tetrahedron. The relative energy of the *bb'* isomer is 9–28 kJ mol<sup>−1</sup>. The energy barriers to intramolecular rearrangements *bb'*( $C_s$ ) → *bb*( $D_{2d}$ ) are also low: 15–35 kJ mol<sup>−1</sup>. These results show that the  $M_2XO_4$  molecules are structurally nonrigid, with a “polytopic” character of the  $M-XO_4$  chemical bonds. The calculation results were compared to the published experimental data on the structure and vibration spectra of the  $M_2XO_4$  molecules.

Molecules of alkali metal sulfates, selenates, chromates, molybdates, and tungstates  $M_2XO_4$  have been studied by gas-phase electron diffraction [1–4] and by Raman and IR spectroscopy using the matrix isolation technique [5–10]. The structural information derived from these studies is far from being unambiguous and exhaustive. All the experimental data were interpreted assuming a single type of equilibrium configurations of nuclei in  $M_2XO_4$  molecules: configurations of the  $D_{2d}$  symmetry in which the  $M^+$  cations coordinate with two opposite O–O edges of a distorted  $XO_4^{2-}$  tetrahedron (Fig. 1a). In a spectroscopic study of  $M_2SO_4$  molecules [5], bistridentate (*tt*) coordination of  $M^+$  ions with the  $SO_4^{2-}$  anion (Fig. 1d) was considered along with the bisbidentate (*bb*) coordination, but in interpretation of the experimental results the bisbidentate ( $D_{2d}$ ) structure was preferred. The possible isomerism of the other  $M_2XO_4$  molecules was discussed in none of the previous papers. As for theoretical studies, only the molecules of lithium, sodium, and potassium sulfates were investigated (see [11, 12] and references therein). No quantum-chemical calculations were performed for the other  $M_2XO_4$  molecules.

Here we report on the first systematic *ab initio* study of  $M_2XO_4$  salt molecules ( $M = \text{Li, Na, K}$ ;  $X = \text{S,$

$\text{Se, Te, Cr, Mo, W}$ ). We have determined their geometries, revealed specific features of their force fields and vibration spectra, and found regular trends in variation of molecular parameters on replacement of  $X$  and  $M$  atoms by their heavier analogs ( $X: \text{S} \rightarrow \text{Se} \rightarrow \text{Te}, \text{Cr} \rightarrow \text{Mo} \rightarrow \text{W}$ ;  $\mu: \text{Li} \rightarrow \text{Na} \rightarrow \text{K}$ ) with the



**Fig. 1.** Alternative configurations of  $M_2XO_4$  molecules: (a) bisbidentate (*bb*),  $D_{2d}$  symmetry; (b) bisbidentate (*bb'*),  $C_s$  symmetry; (c) monobidentate (*mb*),  $C_1$  symmetry; and (d) bistridentate (*tt*),  $C_{2v}$  symmetry.

aim to predict the structure and spectra of heavier  $M_2XO_4$  molecules ( $M = Rb, Cs$ ). The properties of the  $XO_4$  fragment in the  $M_2XO_4$  molecule were compared to those of the free  $XO_4^{2-}$  ion [13]. The minimum-energy pathways of isomerization of  $M_2XO_4$  molecules were studied. The structure and vibration spectra of the molecules of lithium, sodium, and potassium tungstates were studied by us *ab initio* previously [14]; some of those results are used in this work.

**Calculation details.** Calculations were performed using the GAMESS program [15]. The following basis sets of grouped Gaussian functions were used: Li, (9s3p1d)/[4s3p1d]; Na, (12s8p1d)/[6s4p1d]; K, (14s11p3d)/[9s8p3d]; for alkali metal sulfates, also O, (9s5p1d)/[4s2p1d]; S, (12s9p1d)/[6s4p1d]. These basis sets are described in detail in [12]. In calculations of  $M_2XO_4$  molecules with  $X = Se, Te, Cr, Mo$ , and W, we used the effective core potentials by Stevens *et al.* for the Se, Te, Cr, Mo, W, and O atoms [16, 17] (for X atoms the potentials are relativistic). The valence orbitals of the oxygen atom were described by the 31G basis [16] supplemented by the polarization *d* function with an exponent of 0.8. The basis sets of type 41G for the Se and Te atoms [17] were supplemented by two polarization *d* functions [18] with exponents of 0.489 and 0.144 for Se, and 0.305 and 0.096 for Te. The valence orbitals of the Cr, Mo, and W atoms were described by the following basis sets [17]: Cr, (8s8p6d)/[4s4p3d]; Mo, (8s8p5d)/[4s4p3d]; and W, (7s7p5d)/[4s4p3d].

The equilibrium geometries of the  $M_2XO_4$  molecules were optimized, and their vibrational spectra and minimum-energy pathways of intramolecular rearrangements calculated, by the Hartree–Fock (HF) method. The relative energies of alternative configurations were optimized by the method of configuration interactions taking into account single- and double-excited configurations with a Davidson correction [19] for quartic excitations (CISD+Q). The CISD+Q calculations were performed for the geometries optimized on the HF level. When constructing the wave function of the CISD method, we did not take into account only the electron excitations from the molecular orbitals corresponding to the 1s orbitals of O and Na and to the 1s, 2s, and 2p orbitals of S and K. The resulting wave functions consisted of the following numbers of configurations (within the  $D_2$  symmetry): 398 306 ( $Li_2SO_4$ ), 374 581 ( $Li_2SeO_4$ ,  $Li_2TeO_4$ ), 681 583 ( $Li_2CrO_4$ ,  $Li_2MoO_4$ ,  $Li_2WO_4$ ), 736 291 ( $Na_2SO_4$ ), 693 253 ( $Na_2SeO_4$ ,  $Na_2TeO_4$ ), 1 145 341 ( $Na_2CrO_4$ ,  $Na_2MoO_4$ ,  $Na_2WO_4$ ), 1 558 495 ( $K_2SO_4$ ), 1 495 585 ( $K_2SeO_4$ ,  $K_2TeO_4$ ), and 2 327 403 ( $K_2CrO_4$ ,  $K_2MoO_4$ ,  $K_2WO_4$ ).

The geometries of the  $M_2XO_4$  molecules were optimized using the analytically calculated gradients. Calculations of the force constants and dipole moment derivatives with respect to normal coordinates were performed by a numerical method using the procedures and software described in [20].

**Structure of  $M_2XO_4$  molecules.** For each molecule we considered several configurations of nuclei, corresponding to different types of coordination of  $M^+$  cations with tetrahedral anions  $XO_4^{2-}$  (Fig. 1). The optimized geometries, dipole moments, and relative energies of the alternative structures, calculated in the HF approximation, are listed in Tables 1 and 2. Calculations show that the ground equilibrium configuration of all the molecules under consideration is the *bb* structure of the  $D_{2d}$  symmetry (Fig. 1a), corresponding to the bisbidentate coordination of two  $M^+$  cations with the  $XO_4^{2-}$  anion. The bisbidentate structures *bb'* of the  $C_s$  symmetry in which the cations are coordinated at the adjacent O–O edges of the  $XO_4^{2-}$  tetrahedron (Fig. 1b) correspond to local minima on the potential energy surfaces of all the molecules except  $K_2SO_4$ , in which this structure corresponds to the first-order saddle point. Presumably, in  $Rb_2SO_4$  and  $Cs_2SO_4$  the *bb'* structures also correspond to the saddle points on the potential energy surfaces.

Calculations show that the relative energy  $h(bb') = E(bb') - E(bb)$  of the  $C_s$  structures in the series  $Li_2XO_4 \rightarrow Na_2XO_4 \rightarrow K_2XO_4$  varies only slightly and in most cases somewhat increases. If this trend keeps in going to Rb and Cs, for  $Rb_2XO_4$  and  $Cs_2XO_4$  the relative energy  $h(bb')$  will be within 20–30 kJ mol<sup>−1</sup>. It is interesting that the trends in variation of  $h(bb')$  in the series  $M_2SO_4 \rightarrow M_2SeO_4 \rightarrow M_2TeO_4$  and  $M_2CrO_4 \rightarrow M_2MoO_4 \rightarrow M_2WO_4$  are opposite: Whereas in the first case  $h(bb')$  decreases by 3–10 kJ mol<sup>−1</sup>, in the second case it increases by 11–14 kJ mol<sup>−1</sup>.

The results of calculating the minimum-energy pathways of the intramolecular rearrangement  $bb'(C_s) \rightarrow bb(D_{2d})$  in the  $Li_2SO_4$  and  $K_2WO_4$  molecules are shown in Fig. 2. Calculations show that the peaks of the activation barriers of this rearrangement correspond to the monobidentate (*mb*) structures of the  $C_1$  symmetry (Fig. 1a). The height  $h(mb) = E(mb) - E(bb')$  of the barrier separating the local [*bb'(C<sub>s</sub>)*] and global [*bb(D<sub>2d</sub>)*] minima on the potential energy surface is 35 kJ mol<sup>−1</sup> for  $Li_2SO_4$  and 12 kJ mol<sup>−1</sup> for  $K_2WO_4$ . It should be noted that the heights of the activation barriers  $h(mb)$  noticeably exceed the vibrational quanta corresponding to the vibrational motion of nuclei in the *bb'* structures of the  $M_2XO_4$  molecules directed along the most “nonrigid” normal coordinate determining the minimum-energy

**Table 1.** Properties of  $\mu_2\text{XO}_4$  molecules ( $X = \text{S, Se, Te}$ )<sup>a</sup>

Property	Li <sub>2</sub> SO <sub>4</sub>	Na <sub>2</sub> SO <sub>4</sub>	K <sub>2</sub> SO <sub>4</sub>	Li <sub>2</sub> SeO <sub>4</sub>	Na <sub>2</sub> SeO <sub>4</sub>	K <sub>2</sub> SeO <sub>4</sub>	Li <sub>2</sub> TeO <sub>4</sub>	Na <sub>2</sub> TeO <sub>4</sub>	K <sub>2</sub> TeO <sub>4</sub>
<i>bb</i> (D <sub>2d</sub> ) structure									
<i>R</i> <sub>e</sub> (MO)	1.855	2.199	2.523	1.883	2.230	2.559	1.906	2.259	2.595
<i>R</i> <sub>e</sub> (XO)	1.475	1.476	1.476	1.618	1.617	1.617	1.799	1.797	1.796
$\alpha_e$ (OXO)	103.9	106.2	107.2	100.4	103.9	105.4	94.2	98.9	101.2
<i>bb'</i> (C <sub>s</sub> ) structure									
<i>R</i> <sub>e</sub> (XO'')	1.422	1.438	1.451	1.570	1.582	1.590	1.758	1.765	1.770
<i>R</i> <sub>e</sub> (XO')	1.546	1.533	1.519	1.681	1.669	1.658	1.856	1.844	1.834
<i>R</i> <sub>e</sub> (XO)	1.475	1.473	1.470	1.619	1.616	1.614	1.801	1.798	1.796
<i>R</i> <sub>e</sub> (MO')	1.877	2.235	2.596	1.900	2.258	2.616	1.913	2.281	2.648
<i>R</i> <sub>e</sub> (MO)	1.830	2.177	2.510	1.854	2.207	2.536	1.875	2.229	2.557
$\alpha_e$ (O'XO)	101.6	104.5	106.3	98.4	102.4	104.4	92.7	97.8	100.5
$\beta_e$ (O'XO)	113.9	112.7	111.8	114.8	113.4	112.6	116.4	114.8	113.8
$\gamma_e$ (O'XO')	111.7	109.5	107.9	113.8	110.6	109.0	117.8	113.7	111.7
$\theta_e^b$	167.4	158.8	147.1	169.2	162.1	155.2	171.8	167.4	163.6
$\mu_e$	9.5	11.6	11.4	9.6	12.1	12.8	9.2	12.8	14.2
<i>h</i> (HF)	27	27	24	25	26	25	18	24	25
<i>h</i> (CISD+Q)	21	22	–	18	22	–	11	19	–
<i>tt</i> (C <sub>2v</sub> ) structure									
<i>R</i> <sub>e</sub> (XO)	1.453	1.456	1.459	1.580	1.601	1.604	1.763	1.775	1.786
<i>R</i> <sub>e</sub> (XO')	1.492	1.495	1.493	1.668	1.635	1.632	1.857	1.827	1.808
<i>R</i> <sub>e</sub> (MO)	2.016	2.307	2.601	2.671	2.408	2.672	3.073	2.914	2.785
<i>R</i> <sub>e</sub> (MO')	2.207	2.475	2.794	1.986	2.523	2.866	1.961	2.413	2.944
$\alpha_e$ (OXO)	120.0	116.7	114.8	122.9	121.0	117.7	124.5	126.6	124.1
$\beta_e$ (OXO')	108.4	109.1	109.3	109.6	108.4	108.9	110.5	108.3	107.8
$\mu_e$	6.7	9.2	10.5	9.7	9.4	10.7	10.0	11.5	10.8
<i>h</i> (HF)	114	48	21	129	64	32	139	103	63
<i>h</i> (CISD+Q)	95	38	13	109	49	24	120	85	56

<sup>a</sup> The dimensions are as follows: internuclear distances *R*<sub>e</sub>, Å; bond angles  $\alpha_e$ ,  $\beta_e$ ,  $\gamma_e$  and dihedral angles  $\theta_e$ , deg; dipole moments  $\mu_e$ , D; relative energies *h*(*bb'*) = *E*(*bb'*) – *E*(*bb*) and *h*(*tt*) = *E*(*tt*) – *E*(*bb*), kJ mol<sup>–1</sup>. <sup>b</sup> Dihedral angle between the XO<sub>2</sub> and O<sub>2</sub>M planes in the XO<sub>2</sub>M ring. The same for Table 2.

pathway of the rearrangement *bb'*(C<sub>s</sub>) → *bb*(D<sub>2d</sub>). The potential well of the *bb'*(C<sub>s</sub>) structure accommodates 18 (Li<sub>2</sub>SO<sub>4</sub>) and 14 (K<sub>2</sub>WO<sub>4</sub>) vibrational energy levels calculated in the harmonic approximation. Hence, the C<sub>s</sub> structures of the M<sub>2</sub>XO<sub>4</sub> molecules can be experimentally detectable, e.g., at low temperatures using the matrix isolation technique. Nevertheless, very low values of the relative energies of the isomers and of the energy barriers of structural rearrangements of M<sub>2</sub>XO<sub>4</sub> molecules indicate that these molecules are structurally nonrigid, with the “polytopic” character of chemical bonds between the M atoms and XO<sub>4</sub> fragment.

The bistridentate structures *tt* of the C<sub>2v</sub> symmetry (Fig. 1d) correspond to the local minima on the potential energy surface of two molecules: K<sub>2</sub>SO<sub>4</sub> and K<sub>2</sub>CrO<sub>4</sub>. The local (*tt*) and global (*bb*) minima on the potential energy surfaces of these molecules are sep-

arated by a very small energy barrier corresponding to the distorted bistridentate structure *tt'*(C<sub>1</sub>): *h*(*tt'*) = *E*(*tt'*) – *E*(*tt*) = 35 (K<sub>2</sub>SO<sub>4</sub>) or 2 (K<sub>2</sub>CrO<sub>4</sub>) cm<sup>–1</sup>. So small *h*(*tt'*) does not allow the *tt*(C<sub>2v</sub>) structures to be considered as existing isomers of the K<sub>2</sub>SO<sub>4</sub> and K<sub>2</sub>CrO<sub>4</sub> molecules. On the potential energy surfaces of the other M<sub>2</sub>XO<sub>4</sub> molecules under consideration, the bistridentate structures *tt* of the C<sub>2v</sub> symmetry correspond to the saddle points of the first (Li<sub>2</sub>SeO<sub>4</sub>, Li<sub>2</sub>TeO<sub>4</sub>, Li<sub>2</sub>CrO<sub>4</sub>, Li<sub>2</sub>MoO<sub>4</sub>, Li<sub>2</sub>WO<sub>4</sub>, Na<sub>2</sub>TeO<sub>4</sub>, K<sub>2</sub>SeO<sub>4</sub>), second (Li<sub>2</sub>SO<sub>4</sub>, Na<sub>2</sub>SO<sub>4</sub>, Na<sub>2</sub>SeO<sub>4</sub>, Na<sub>2</sub>CrO<sub>4</sub>, K<sub>2</sub>TeO<sub>4</sub>, K<sub>2</sub>MoO<sub>4</sub>, K<sub>2</sub>WO<sub>4</sub>), and third orders (Na<sub>2</sub>MoO<sub>4</sub> and Na<sub>2</sub>WO<sub>4</sub>). The relative energy of the *tt* structures *h*(*tt*) = *E*(*tt*) – *E*(*bb*) of the M<sub>2</sub>XO<sub>4</sub> molecules appreciably (by 25–47 kJ mol<sup>–1</sup>) decreases with decreasing weight of the central atom X (X = Te → Se → S, W → Mo → Cr).

The energies *h*(*tt*) decrease in the series Li<sub>2</sub>XO<sub>4</sub> →

**Table 2.** Properties of  $M_2XO_4$  molecules ( $X = Cr, Mo, W$ )

Property	$Li_2CrO_4$	$Na_2CrO_4$	$K_2CrO_4$	$Li_2MoO_4$	$Na_2MoO_4$	$K_2MoO_4$	$Li_2WO_4$	$Na_2WO_4$	$K_2WO_4$
<i>bb</i> ( $D_{2d}$ ) structure									
$R_e(MO)$	1.858	2.223	2.538	1.898	2.264	2.586	1.910	2.270	2.592
$R_e(XO)$	1.604	1.605	1.605	1.753	1.754	1.754	1.763	1.763	1.763
$\alpha_e(OXO)$	101.6	105.0	105.9	96.6	101.0	102.6	96.5	101.0	102.5
<i>bb'</i> ( $C_s$ ) structure									
$R_e(XO'')$	1.533	1.548	1.556	1.687	1.698	1.706	1.706	1.716	1.722
$R_e(XO')$	1.699	1.684	1.670	1.836	1.822	1.808	1.828	1.818	1.805
$R_e(XO)$	1.604	1.602	1.602	1.758	1.756	1.756	1.769	1.766	1.766
$R_e(MO')$	1.849	2.229	2.571	1.901	2.286	2.640	1.929	2.308	2.666
$R_e(MO)$	1.869	2.226	2.532	1.891	2.247	2.554	1.884	2.242	2.550
$\alpha_e(O'XO)$	100.3	104.5	105.7	95.4	100.7	102.6	95.4	100.5	102.4
$\beta_e(O'XO)$	113.0	111.5	111.1	114.7	112.7	112.1	115.0	113.1	112.4
$\gamma_e(O'XO')$	112.9	110.0	109.3	115.6	112.2	110.9	114.8	111.7	110.6
$\theta_e$	171.6	163.9	160.8	173.2	168.1	165.5	173.2	168.1	165.7
$\mu_e$	8.5	11.3	12.6	8.8	12.2	13.8	9.2	12.6	14.2
$h(HF)$	11	12	17	18	20	23	26	26	28
$h(CISD+Q)$	9	10	–	16	17	–	23	24	–
<i>tt</i> ( $C_{2v}$ ) structure									
$R_e(XO)$	1.545	1.584	1.588	1.694	1.787	1.750	1.714	1.794	1.765
$R_e(XO')$	1.683	1.626	1.623	1.840	1.722	1.756	1.833	1.732	1.760
$R_e(MO)$	2.777	2.391	2.664	3.139	2.076	2.652	3.122	2.060	2.616
$R_e(MO')$	1.948	2.503	2.837	1.957	3.715	3.065	1.978	3.886	3.145
$\alpha_e(OXO)$	116.2	119.1	116.7	115.0	114.5	120.8	117.2	113.5	121.1
$\beta_e(OXO')$	111.7	108.9	109.2	113.2	108.7	108.3	112.5	108.8	108.1
$\mu_e$	9.1	9.4	10.7	9.9	1.5	10.0	10.6	0.7	9.5
$h(HF)$	115	49	33	144	88	64	159	85	68
$h(CISD+Q)$	104	36	28	136	88	59	147	91	60

$Na_2XO_4 \rightarrow K_2XO_4$ , which suggests that for  $Rb_2XO_4$  and  $Cs_2XO_4$  the relative energies of the *tt* structures will be still lower. To estimate  $h(tt)$  for  $Rb_2XO_4$  and  $Cs_2XO_4$ , we postulated the following relationship between  $h(tt)$  and the ionic radius of alkali metal  $R = R(M^+)$ :

$$h = a + b/R + c/R^2. \quad (1)$$

Extrapolation by formula (1) of the relative energies of the *tt* structures of  $M_2XO_4$  ( $M = Li, Na, K$ ), calculated in the CISD+Q approximation, was performed using the ionic radii evaluated in [21] from the internuclear distances in diatomic alkali metal halide molecules (Table 1, model 1 in [21]). The following values of  $h(tt)$  (kJ mol<sup>-1</sup>) were obtained for  $Rb_2XO_4$  and  $Cs_2XO_4$ :

X =	S	Se	Te	Cr	Mo	W
M = Rb	6	17	47	28	51	51
M = Cs	1	12	39	28	43	44

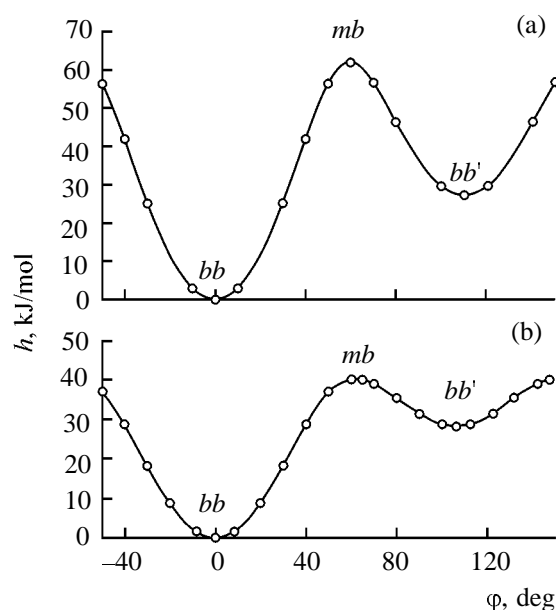
Hence, it can be expected that, similar to the molecules of  $M_2XO_4$  salts of light alkali metals ( $M = Li, Na, K$ ), the ground configuration of nuclei in the  $Rb_2XO_4$  and  $Cs_2XO_4$  molecules will be that of the  $D_{2d}$  symmetry. However, the nearly zero  $h(tt)$  values for the rubidium and cesium sulfate molecules show that in these molecules the cations virtually freely migrate around the  $SO_4^{2-}$  ion, i.e., the structural nonrigidity of these molecules is strongly pronounced. Estimations of  $h(tt)$  for the rubidium and cesium salts  $M_2SO_4$  also show that in the sulfates and selenates the energy of the *tt* structures should be lower than that of the bis-bidentate *bb'* structures. It was interesting to check these assumptions by performing direct *ab initio* calculations of the  $Rb_2XO_4$  and  $Cs_2XO_4$  molecules.

The Mulliken population analysis of  $M_2XO_4$  molecules shows that the charges on alkali metal atoms  $q(M)$  are close to unity, i.e., the chemical bonds between the  $XO_4$  fragment and the metal atoms are essentially polar. The low equilibrium internuclear distances  $R_e(X-O)$  in the  $M_2XO_4$  molecules are close

to  $R_e(X-O)$  in free  $XO_4^{2-}$  ions ( $T_d$  symmetry) [13]. The angles between the  $X-O$  bonds in the  $bb$ ,  $bb'$ , and  $tt$  structures, as a rule, somewhat differ from the ideal tetrahedral angle ( $109^\circ 28'$ ): the mean deviation is  $6^\circ$ , and the largest deviation ( $17^\circ$ ) is observed for the  $\alpha_e(OXO)$  bond angle in the  $tt$  structure of the  $Na_2TeO_4$  molecule. Hence, the structure of the  $M_2XO_4$  molecules can be approximately described by the model  $(M^+)_2[XO_4]^{2-}$ , in which the  $XO_4$  fragment of the  $M_2XO_4$  molecule is considered as an  $XO_4^{2-}$  anion electronically polarized and geometrically distorted in the field of two  $M^+$  cations. The degree of polarization and deformation of the  $XO_4^{2-}$  ion decreases in going from lighter to heavier alkali metals  $M$  ( $Li_2XO_4 \rightarrow Na_2XO_4 \rightarrow K_2XO_4$ ), which is due to a decrease in the polarizing power of the  $M^+$  ion with an increase in its ionic radius. The degree and character of the polarization and deformation of the  $XO_4^{2-}$  ion noticeably change as the position of the cations relative to the anion is changed. In going from the  $bb$  structure to less symmetrical configurations, the  $XO_4^{2-}$  fragment becomes appreciably more distorted (Tables 1, 2). The equilibrium internuclear distances  $R_e(X-O)$  noticeably increase in the series  $M_2SO_4 \rightarrow M_2SeO_4 \rightarrow M_2TeO_4$  (for the  $D_{2d}$  structures, by  $0.14 \text{ \AA}$  in going from S to Se and by  $0.18 \text{ \AA}$  in going from Se to Te). In the series  $M_2CrO_4 \rightarrow M_2MoO_4 \rightarrow M_2WO_4$ , the pattern is somewhat different: in going from chromates to molybdates  $R_e(X-O)$  increases by  $0.15 \text{ \AA}$ , whereas in going from molybdates to tungstates it increases by only  $0.01 \text{ \AA}$ , which is due to lanthanide contraction.

The results of our calculations demonstrate essential similarity in the structures of alkali metal salts with oxyanions of Group VI elements, both  $d$  [ $M_2X^{(d)}O_4$ ,  $X^{(d)} = Cr, Mo, W$ ] and  $p$  [ $M_2X^{(p)}O_4$ ,  $X^{(p)} = S, Se, Te$ ]. The parameters of the  $M_2CrO_4$  molecules are closer to those of the  $M_2SeO_4$  molecules, and the parameters of the  $M_2MoO_4$  and  $M_2WO_4$  molecules, to those of the  $M_2TeO_4$  molecules.

To assess how consideration of the electron correlation affects the results of calculations of the structures and vibration spectra of  $M_2XO_4$  molecules, we have performed complete optimization of geometry by the CISD and CISD+Q methods for the ground isomer  $bb(D_{2d})$  and by the CISD method, for the excited isomer  $bb'(C_s)$  of the  $Li_2SO_4$  molecule. In the optimization in the CISD approximation, we used the analytical gradients. The equilibrium geometries of the ground isomer of the molecule, calculated in the CISD+Q and CISD (in parentheses) approximations, are as follows:  $R_e(Li-O)$  1.863 (1.858),  $R_e(S-O)$  1.503 (1.490)  $\text{\AA}$ ,  $\alpha_e(OSO)$   $104.3^\circ$  ( $104.2^\circ$ ). The geometries of the  $bb'(C_s)$  isomer calculated by the CISD



**Fig. 2.** Minimum-energy pathways of the intramolecular rearrangement  $bb(D_{2d}) \rightarrow mb(C_1) \rightarrow bb'(C_s)$  in (a)  $Li_2SO_4$  and (b)  $K_2WO_4$  molecules. The reaction coordinate  $\varphi$  is the dihedral angle describing deviation of the  $M$  atom from the  $XO_2$  plane.

method are as follows:  $R_e(S-O')$  1.435,  $R_e(S-O'')$  1.568,  $R_e(S-O)$  1.490,  $R_e(Li-O')$  1.871,  $R_e(Li-O)$  1.836  $\text{\AA}$ ,  $\alpha_e(O'SO)$   $101.6^\circ$ ,  $\beta_e(O''SO)$   $114.0^\circ$ ,  $\gamma_e(O'SO)$   $111.5^\circ$ , and  $\theta_e$   $166.9^\circ$ . Thus, taking into account the electron correlation in the computations increases the calculated internuclear distances  $R_e(S-O)$  in the ground and excited isomers of the  $Li_2SO_4$  molecule by  $0.012$ – $0.028 \text{ \AA}$ , whereas the  $R_e(Li-O)$  distances change by no more than  $0.008 \text{ \AA}$ . The  $\alpha_e$ ,  $\beta_e$ ,  $\gamma_e$ , and  $\theta_e$  angles change by no more than  $0.5^\circ$  when the electron correlation is taken into account. Similar correlation corrections to the molecular geometries can be also expected for the other  $M_2XO_4$  molecules. The force constant matrices calculated by the CISD and CISD+Q methods confirmed the conclusion, following from the Hartree-Fock calculations, that the  $Li_2SO_4$  molecule can exist in the form of two isomers with the  $D_{2d}$  and  $C_s$  symmetry. The relative energy  $h(bb')$  calculated in the CISD and CISD+Q approximations using the geometries of the  $bb(D_{2d})$  and  $bb'(C_s)$  structures optimized in the CISD approximation amounted to  $22.8$  (CISD) and  $20.6 \text{ kJ mol}^{-1}$  (CISD+Q). This values differ, respectively, by only  $0.2$  and  $0.5 \text{ kJ mol}^{-1}$  from the  $h(bb')$  values calculated using the HF-optimized geometries:  $h(\text{CISD})$   $23.0$  and  $h(\text{CISD+Q})$   $21.1 \text{ kJ mol}^{-1}$ . This result suggests that for the relative energies of the  $bb'$  configurations of the other  $M_2XO_4$  molecules the error originating from

**Table 3.** Vibration frequencies ( $\omega_i$ ,  $\text{cm}^{-1}$ ) and IR intensities ( $A_i$ ,  $\text{km mol}^{-1}$ , in parentheses) for  $\text{M}_2\text{XO}_4$  molecules ( $\text{X} = \text{S}, \text{Se}, \text{Te}$ )

Parameter	$\text{Li}_2\text{SO}_4$	$\text{Na}_2\text{SO}_4$	$\text{K}_2\text{SO}_4$	$\text{Li}_2\text{SeO}_4$	$\text{Na}_2\text{SeO}_4$	$\text{K}_2\text{SeO}_4$	$\text{Li}_2\text{TeO}_4$	$\text{Na}_2\text{TeO}_4$	$\text{K}_2\text{TeO}_4$
<i>bb</i> ( $D_{2d}$ ) structure									
$\omega_1$ ( $A_1$ )	1060	1054	1057	914	908	910	818	813	815
$\omega_2$ ( $A_1$ )	668	575	544	611	488	452	573	422	382
$\omega_3$ ( $A_1$ )	499	281	201	444	263	189	400	250	179
$\omega_4$ ( $B_2$ )	1276 (526)	1250 (604)	1241 (700)	995 (223)	978 (279)	975 (319)	874 (161)	864 (202)	863 (228)
$\omega_5$ ( $B_2$ )	758 (81)	729 (76)	706 (106)	590 (471)	552 (151)	524 (182)	557 (448)	452 (193)	420 (219)
$\omega_6$ ( $B_2$ )	612 (418)	360 (140)	279 (133)	568 (44)	323 (101)	246 (94)	477 (64)	297 (74)	223 (70)
$\omega_7$ ( $B_1$ )	418	439	450	309	330	340	228	244	253
$\omega_8$ ( $E$ )	1207 (915)	1205 (966)	1207 (875)	946 (440)	951 (460)	953 (417)	834 (316)	839 (324)	842 (301)
$\omega_9$ ( $E$ )	668 (163)	666 (123)	669 (115)	511 (269)	482 (200)	479 (185)	440 (312)	378 (271)	368 (251)
$\omega_{10}$ ( $E$ )	410 (21)	270 (10)	213 (10)	361 (5)	250 (1)	197 (3)	319 (83)	227 (3)	177 (0)
$\omega_{11}$ ( $E$ )	159 (191)	69 (123)	42 (93)	154 (139)	67 (97)	41 (75)	151 (61)	73 (68)	45 (57)
<i>bb'</i> ( $C_s$ ) structure									
$\omega_1$ ( $A'$ )	1405 (414)	1344 (442)	1298	1060 (153)	1032 (174)	1016 (183)	898 (85)	890 (104)	886 (117)
$\omega_2$ ( $A'$ )	1105 (254)	1114 (316)	1130	915 (60)	917 (76)	922 (77)	815 (30)	816 (41)	819 (45)
$\omega_3$ ( $A'$ )	989 (299)	1001 (253)	1024	842 (252)	848 (229)	864 (188)	764 (228)	770 (198)	781 (165)
$\omega_4$ ( $A'$ )	717 (102)	692 (53)	679	612 (259)	520 (120)	497 (117)	578 (241)	434 (161)	402 (159)
$\omega_5$ ( $A'$ )	658 (73)	660 (70)	665	538 (8)	477 (104)	477 (93)	476 (59)	376 (137)	368 (124)
$\omega_6$ ( $A'$ )	619 (122)	482 (2)	483	491 (146)	374 (0)	373 (0)	418 (185)	303 (4)	282 (2)
$\omega_7$ ( $A'$ )	438 (12)	320 (50)	235	337 (11)	288 (34)	216 (29)	292 (15)	252 (26)	202 (25)
$\omega_8$ ( $A'$ )	369 (24)	187 (17)	108	320 (0)	179 (8)	112 (9)	246 (8)	168 (1)	112 (4)
$\omega_9$ ( $A'$ )	193 (91)	80 (46)	32	189 (61)	83 (35)	47 (23)	184 (21)	93 (22)	57 (16)
$\omega_{10}$ ( $A''$ )	1239 (494)	1242 (543)	1252	966 (222)	966 (253)	970 (266)	848 (162)	850 (183)	853 (192)
$\omega_{11}$ ( $A''$ )	722 (126)	705 (73)	691	614 (260)	546 (123)	514 (152)	576 (218)	464 (130)	419 (159)
$\omega_{12}$ ( $A''$ )	640 (96)	533 (3)	503	547 (5)	421 (2)	401 (0)	486 (46)	343 (12)	319 (14)
$\omega_{13}$ ( $A''$ )	480 (81)	323 (42)	261	423 (55)	295 (11)	238 (12)	397 (31)	258 (18)	210 (14)
$\omega_{14}$ ( $A''$ )	444 (24)	303 (50)	235	377 (20)	275 (56)	212 (53)	295 (44)	253 (44)	187 (36)
$\omega_{15}$ ( $A''$ )	157 (10)	56 (0)	51 <i>i</i>	162 (8)	68 (0)	19 (0)	178 (3)	89 (1)	57 (0)

the use of the Hartree–Fock equilibrium geometries in the CISD+Q calculations will also be insignificant.

Comparison of the relative energies  $h(bb')$  and  $h(tt)$  calculated by the HF and CISD+Q methods (Tables 1, 2) shows that in most cases taking into account the electron correlation decreases the relative energy of the “excited” configurations (by 2–20  $\text{kJ mol}^{-1}$ ). The exceptions are the  $tt$  configurations of the  $\text{Na}_2\text{MoO}_4$  and  $\text{Na}_2\text{WO}_4$  molecules, for which  $h(tt)$  remains unchanged or slightly increases when the electron correlation is taken into account. It should be noted that for molecules with low relative energies of “excited” configurations the CI calculations give appreciably decreased values of  $h$ . For example, in going from HF to CISD+Q calculations, for  $\text{K}_2\text{SO}_4$   $h(tt) = E(tt) - E(bb)$  decreases from 21 to 13  $\text{kJ mol}^{-1}$ .

**Force fields and vibration spectra of  $\text{M}_2\text{XO}_4$  molecules.** The calculated vibration frequencies and intensities in the IR spectra of  $\text{M}_2\text{XO}_4$  molecules are

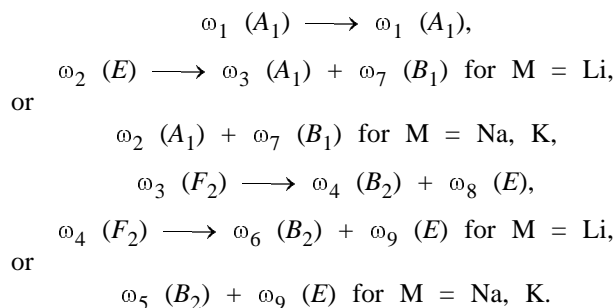
listed in Tables 3 and 4. The vibrational representations of the  $\text{M}_2\text{XO}_4$  molecule with the structures of the  $D_{2d}$  and  $C_s$  symmetry are  $3A_1 + B_1 + 3B_2 + 4E$  and  $9A' + 6A''$ , respectively. We used the system of vibrational coordinates of the  $\text{M}_2\text{XO}_4$  molecules described in [14].

Analysis of the normal modes and distribution of the vibrational potential energy throughout the vibrational coordinates shows that many vibrations of the  $\text{M}_2\text{XO}_4$  molecules are appreciably mixed (to a greater extent in  $\text{Li}_2\text{XO}_4$  and to a lesser extent in  $\text{K}_2\text{XO}_4$ ) and therefore cannot be interpreted unambiguously. Nevertheless, we can conventionally distinguish nine normal modes of the  $\text{M}_2\text{XO}_4$  molecule correlating with vibrations of the  $\text{XO}_4^{2-}$  tetrahedron (vibrational representation  $A_1 + E + 2F_2$ ). The remaining six modes correspond to motion of the  $\text{M}^+$  cations relative to the  $\text{XO}_4^{2-}$  anion. The correlations between the vibrations of the free  $\text{XO}_4^{2-}$  ion and those of the  $\text{XO}_4^{2-}$  ion in the

**Table 4.** Vibration frequencies ( $\omega_i$ ,  $\text{cm}^{-1}$ ) and IR intensities ( $A_i$ ,  $\text{km mol}^{-1}$ , in parentheses) for  $M_2XO_4$  molecules ( $X = \text{Cr, Mo, W}$ )

Parameter	$\text{Li}_2\text{CrO}_4$	$\text{Na}_2\text{CrO}_4$	$\text{K}_2\text{CrO}_4$	$\text{Li}_2\text{MoO}_4$	$\text{Na}_2\text{MoO}_4$	$\text{K}_2\text{MoO}_4$	$\text{Li}_2\text{WO}_4$	$\text{Na}_2\text{WO}_4$	$\text{K}_2\text{WO}_4$
<i>bb(D<sub>2d</sub>)</i> structure									
$\omega_1 (A_1)$	1036	1024	1027	1002	994	995	1027	1019	1022
$\omega_2 (A_1)$	622	485	459	580	432	405	579	435	410
$\omega_3 (A_1)$	460	270	200	428	262	192	436	264	193
$\omega_4 (B_2)$	1041 (639)	1014 (733)	1011 (815)	942 (723)	923 (778)	919 (827)	924 (614)	908 (647)	906 (682)
$\omega_5 (B_2)$	603 (421)	523 (18)	507 (18)	574 (411)	449 (62)	427 (66)	573 (397)	436 (95)	413 (103)
$\omega_6 (B_2)$	571 (95)	335 (153)	264 (143)	492 (96)	310 (124)	239 (110)	472 (83)	295 (92)	224 (82)
$\omega_7 (B_1)$	364	375	379	309	318	322	308	319	324
$\omega_8 (E)$	948 (933)	986 (1059)	987 (980)	888 (894)	899 (996)	898 (933)	861 (514)	887 (836)	889 (787)
$\omega_9 (E)$	456 (27)	438 (4)	437 (3)	390 (124)	361 (52)	359 (45)	386 (138)	346 (87)	343 (77)
$\omega_{10} (E)$	386 (2)	254 (2)	207 (3)	344 (1)	224 (0)	181 (1)	313 (2)	218 (0)	177 (1)
$\omega_{11} (E)$	170 (157)	67 (107)	47 (82)	168 (86)	75 (85)	51 (67)	158 (44)	72 (77)	49 (59)
<i>bb'(C<sub>s</sub>)</i> structure									
$\omega_1 (A')$	1194 (400)	1145 (406)	1122 (403)	1079 (268)	1053 (245)	1038 (214)	1076 (132)	1056 (110)	1047 (87)
$\omega_2 (A')$	1026 (165)	1017 (189)	1018 (184)	958 (288)	950 (326)	947 (352)	953 (340)	944 (368)	941 (392)
$\omega_3 (A')$	837 (426)	832 (494)	855 (470)	790 (446)	789 (496)	806 (471)	800 (387)	796 (417)	813 (396)
$\omega_4 (A')$	612 (174)	469 (16)	460 (10)	582 (168)	409 (60)	393 (45)	580 (172)	405 (78)	386 (62)
$\omega_5 (A')$	504 (24)	432 (4)	434 (3)	452 (59)	361 (33)	356 (30)	445 (57)	350 (43)	343 (39)
$\omega_6 (A')$	434 (17)	397 (10)	395 (4)	378 (63)	330 (6)	331 (1)	372 (87)	320 (11)	326 (14)
$\omega_7 (A')$	367 (1)	303 (39)	232 (40)	312 (1)	285 (28)	218 (32)	311 (3)	278 (24)	209 (24)
$\omega_8 (A')$	351 (2)	188 (10)	127 (10)	288 (3)	166 (6)	115 (7)	262 (2)	153 (5)	109 (5)
$\omega_9 (A')$	199 (66)	85 (37)	57 (22)	187 (33)	91 (26)	60 (17)	174 (28)	88 (24)	57 (15)
$\omega_{10} (A'')$	1044 (572)	1027 (639)	1021 (669)	928 (577)	917 (637)	912 (652)	904 (495)	897 (541)	895 (553)
$\omega_{11} (A'')$	615 (245)	514 (27)	486 (22)	569 (234)	448 (59)	415 (60)	567 (204)	445 (64)	411 (68)
$\omega_{12} (A'')$	534 (15)	435 (16)	425 (16)	486 (11)	363 (1)	356 (0)	479 (24)	350 (2)	346 (6)
$\omega_{13} (A'')$	423 (0)	300 (46)	239 (16)	371 (2)	278 (50)	219 (39)	362 (25)	277 (39)	218 (25)
$\omega_{14} (A'')$	411 (5)	289 (22)	235 (60)	347 (7)	251 (4)	197 (20)	331 (3)	236 (12)	181 (24)
$\omega_{15} (A'')$	182 (14)	72 (1)	47 (0)	198 (10)	92 (2)	66 (0)	193 (7)	92 (1)	66 (0)

ground isomer *bb(D<sub>2d</sub>)* of the  $M_2XO_4$  molecules are shown below:



The strongest bands in the IR spectra of  $M_2XO_4$  molecules are those originating from vibrations of the  $XO_4$  fragment, and the weakest bands originate from the motion of  $M^+$  cations relative to the  $XO_4^{2-}$  anion.

In the series  $\text{Li}_2XO_4 \rightarrow \text{Na}_2XO_4 \rightarrow \text{K}_2XO_4$ , the force constants of M–O stretching and  $XO_2M$  out-of-

plane bending (along the coordinate  $\chi$  [12]) considerably decrease. For example, the constant  $f_\chi$  for the  $M_2SO_4$  molecules of the *bb(D<sub>2d</sub>)* structure decreases from 0.078 mdyne Å for  $M = \text{Li}$  to 0.027 mdyne Å for  $M = \text{K}$ . At the same time, the force constants describing the dynamics of the  $XO_4$  fragment vary less significantly. Similar trends were also observed with the corresponding vibration frequencies (Tables 3, 4).

The possible CI corrections to the Hartree–Fock frequencies and IR intensities for the  $M_2XO_4$  molecules can be evaluated by comparing the results of calculating these quantities by the HF (Table 3) and CISD+Q methods. The vibrational frequencies  $\omega_i$  ( $\text{cm}^{-1}$ ) found in the CISD+Q approximation and the IR intensities  $A_i$  ( $\text{km mol}^{-1}$ , found in the CISD approximation, given in parentheses) for the ground isomer *bb(D<sub>2d</sub>)* of the  $\text{Li}_2\text{SO}_4$  molecule are as follows ( $i = 1\text{--}11$ ): 968, 649, 466, 1184 (469), 697 (93), 600

**Table 5.** Vibration frequencies ( $\nu_i$ ,  $\text{cm}^{-1}$ ) of matrix-isolated  $\text{M}_2\text{XO}_4$  molecules

Molecule	$\nu_1$ ( $A_1$ )	$\nu_4$ ( $B_2$ )	$\nu_5$ ( $B_2$ )	$\nu_6$ ( $B_2$ )	$\nu_8$ ( $E$ )	$\nu_9$ ( $E$ )	$\nu_{10}$ ( $E$ )
$\text{Li}_2\text{SO}_4$	—	1140 <sup>a</sup>	—	—	1101 <sup>a</sup>	—	—
$\text{Na}_2\text{SO}_4$	961 <sup>b</sup>	1134–1137 <sup>c</sup>	640 <sup>b</sup>	—	1100 <sup>c</sup>	610 <sup>b</sup>	295 <sup>c</sup>
$\text{K}_2\text{SO}_4$ <sup>d</sup>	962 <sup>b</sup>	1128 <sup>c</sup>	640 <sup>c</sup>	262 <sup>c</sup>	1099 <sup>c</sup>	608 <sup>c</sup>	220–240 <sup>c</sup>
$\text{Rb}_2\text{SO}_4$	—	1122–1128 <sup>c</sup>	636 <sup>c</sup>	215 <sup>c</sup>	1096–1100 <sup>c</sup>	608 <sup>c</sup>	190–200 <sup>c</sup>
$\text{Cs}_2\text{SO}_4$	962 <sup>e</sup>	1125 <sup>c</sup>	631–635 <sup>c</sup>	196 <sup>c</sup>	1090–1100 <sup>c</sup>	610 <sup>c</sup>	148–170 <sup>c</sup>
$\text{M}_2\text{SeO}_4$ <sup>f</sup>	—	880–881	377–389	—	894	438–445	—
$\text{M}_2\text{CrO}_4$ <sup>g</sup>	—	895–898	—	—	877	—	—
$\text{K}_2\text{CrO}_4$	853 <sup>e</sup>	893 <sup>h</sup>	429 <sup>h</sup>	248 <sup>h</sup>	875 <sup>h</sup>	—	—
$\text{Rb}_2\text{CrO}_4$	—	890 <sup>i</sup>	—	—	877 <sup>i</sup>	—	—
$\text{Cs}_2\text{CrO}_4$	847 <sup>e</sup>	889 <sup>i</sup>	—	—	876 <sup>i</sup>	—	—
$\text{M}_2\text{MoO}_4$ <sup>g</sup>	—	840–844	—	—	830	—	—
$\text{K}_2\text{MoO}_4$	886 <sup>e</sup>	839 <sup>h</sup>	378 <sup>h</sup>	227 <sup>h</sup>	827 <sup>h</sup>	318 <sup>h</sup>	—
$\text{Rb}_2\text{MoO}_4$	—	839 <sup>i</sup>	—	—	830 <sup>i</sup>	—	—
$\text{Cs}_2\text{MoO}_4$	891 <sup>e</sup>	839 <sup>a</sup>	—	—	830 <sup>a</sup>	—	—
$\text{Li}_2\text{WO}_4$	—	839 <sup>a</sup>	—	—	830 <sup>a</sup>	—	—
$\text{Na}_2\text{WO}_4$	—	836 <sup>a</sup>	376 <sup>j</sup>	—	830 <sup>a</sup>	306 <sup>j</sup>	—
$\text{K}_2\text{WO}_4$	925 <sup>e</sup>	836 <sup>a</sup>	351 <sup>a</sup>	218 <sup>h</sup>	831 <sup>a</sup>	306 <sup>a</sup>	—
$\text{Rb}_2\text{WO}_4$	—	836 <sup>i</sup>	—	—	830 <sup>i</sup>	—	—
$\text{Cs}_2\text{WO}_4$	925 <sup>e</sup>	835 <sup>a</sup>	—	—	830 <sup>a</sup>	—	—

<sup>a</sup> IR spectrum ( $\text{N}_2$  matrix) [6]. <sup>b</sup> Raman spectrum ( $\text{N}_2$ ) [6]. <sup>c</sup> IR spectrum (Ar) [5]. <sup>d</sup> IR spectrum [5]:  $\nu_1$  ( $E$ ) 70–82 (Xe), 65–85 (Kr), and 57–67 (Ar)  $\text{cm}^{-1}$ . <sup>e</sup> Raman spectrum (Ne) [9]. <sup>f</sup> M = Na, K, Rb, Cs; IR spectrum ( $\text{N}_2$ ) [10]. <sup>g</sup> M = Li, Na; IR spectrum ( $\text{N}_2$ ) [6]. <sup>h</sup> IR spectrum (Ar) [7]. <sup>i</sup> IR spectrum ( $\text{N}_2$ ) [8]. <sup>j</sup> IR spectrum (Ar) [6].

(400), 373, 1100 (820), 613 (144), 409 (10), and 147 (185). Thus, taking into account electron correlation decreases by 0.2–11% the frequencies  $\omega_i$  and only weakly affects the IR intensities.

**Comparison of the results of *ab initio* calculations with the experiment.** The results of mass-spectrometric study of the vapors of alkali metal sulfates [22], selenates [10], chromates [22, 23], molybdates [22], and tungstates [22, 24] in a wide temperature range (800–2000 K) show that these compounds vaporize without significant decomposition and exist in the gas phase mainly as monomeric molecules  $\text{M}_2\text{XO}_4$ .

Let us now compare the results of *ab initio* calculations of the vibration spectra of  $\text{M}_2\text{XO}_4$  molecules with the experimental IR and Raman spectra of species obtained by vaporization of  $\text{M}_2\text{XO}_4$  salts and isolated in inert matrices [5–10]. The experimental frequencies [5–10], listed in Table 5, are consistent with the bisbidentate ( $D_{2d}$ ) structure of the  $\text{M}_2\text{XO}_4$  molecules. Alkali metal sulfates have been studied most extensively; e.g., for  $\text{K}_2\text{SO}_4$  the frequencies of all the IR-active normal modes and of the Raman-active  $\nu_1$  ( $A_1$ ) mode are known from the experiment. Data on vibration spectra of the  $\text{M}_2\text{TeO}_4$  and  $\text{Li}_2\text{SeO}_4$  molecules are lacking. For the majority of the other  $\text{M}_2\text{XO}_4$  molecules, only the frequencies of the  $\text{XO}_4$

fragments are known; the results of *ab initio* calculations agree with these data. The IR spectra of  $\text{K}_2\text{XO}_4$  molecules ( $\text{X} = \text{Cr}, \text{Mo}, \text{W}$ ) isolated in an argon matrix contained in the low-frequency range corresponding to metal–oxygen stretching vibrations a single band [7] assignable, according to our calculations, to the  $\omega_6$  ( $B_2$ ) K–O vibrations of the  $D_{2d}$  isomer of the  $\text{K}_2\text{XO}_4$  molecules.

The vibration frequencies of the  $\text{M}_2\text{XO}_4$  molecules calculated by us systematically exceed the experimental frequencies: by 3–10% for stretching modes and by 12–15% for bending modes. Such a deviation from the experiment is due, on the one hand, to insufficient completeness of the set of basis functions used in our calculations, possible defects of the effective core potentials, and neglect of the electron correlation and of the anharmonicity effects and, on the other hand, to certain difference between the vibration frequencies of matrix-isolated and free molecules.

In view of the results of our study, interpretation of the vibration spectra of  $\text{M}_2\text{XO}_4$  molecules made in spectroscopic studies [5–10] assuming existence of a single structure of these molecules seems to be incomplete. In the IR spectra of  $\text{M}_2\text{XO}_4$  molecules a number of additional bands were observed [5, 7], both in the range of X–O stretching vibrations and in the low-



**Table 6.** Apparent internuclear distances and bond angles in the  $M_2XO_4$  molecules, determined by high-temperature gas-phase electron diffraction

Molecule	$R_g(MO)$ , Å	$R_g(XO)$ , Å	$\alpha_g(OXO)$ , deg	Vapor temperature, K	References
$K_2SO_4$	$2.45 \pm 0.03$	$1.47 \pm 0.01$	$109 \pm 10$	$1670 \pm 100$	[1]
$Cs_2SO_4$	$2.60 \pm 0.03$	$1.48 \pm 0.01$	$109 \pm 10$	$1670 \pm 100$	[1]
$Cs_2SO_4$	$2.80 \pm 0.05$	$1.471 \pm 0.004$	$109 \pm 4$	1320	[3]
$K_2CrO_4$	$2.45 \pm 0.03$	$1.66 \pm 0.01$	$109 \pm 10$	$1370 \pm 100$	[1]
$Cs_2CrO_4$	$2.85 \pm 0.04$	$1.675 \pm 0.006$	$108 \pm 2$	$1470 \pm 50$	[4]
$Rb_2MoO_4$	$2.74 \pm 0.02$	$1.779 \pm 0.009$	$101 \pm 2$	$1170 \pm 30$	[2]
$Cs_2MoO_4$	$2.80 \pm 0.03$	$1.80 \pm 0.02$	$105 \pm 4$	$1320 \pm 30$	[2]
$Rb_2WO_4$	$2.69 \pm 0.02$	$1.798 \pm 0.009$	$104 \pm 2$	$1070 \pm 30$	[2]
$Cs_2WO_4$	$2.78 \pm 0.04$	$1.82 \pm 0.02$	$104 \pm 4$	$1170 \pm 30$	[2]

frequency range. These bands were not unambiguously assigned in [5, 7]. Our results suggest that some of these bands can be assigned to vibrations of the  $XO_4$  fragments in the  $C_s$  isomers of the  $M_2XO_4$  molecules.

The molecules of potassium, rubidium, and cesium sulfates, chromates, molybdates, and tungstates were studied by high-temperature gas-phase electron diffraction in [1–4] (Table 6). Before comparing the results of our calculations with those of electron diffraction studies, three facts should be noted.

(1) The error in calculating the equilibrium internuclear distances, originating from insufficiently high level of the theory used (incompleteness of basis sets, neglect of electron correlation, possible defects of the effective core potentials) can be significant (up to  $0.02$ – $0.03$  Å). At the same time, the error in determining the relative geometries can be much lower, because calculations for all the molecules are performed on the same level of theory using similar effective core potentials and basis sets. Therefore, we believe that the trends in variation of the molecular parameters in the series  $Li_2XO_4 \rightarrow Na_2XO_4 \rightarrow K_2XO_4$ ,  $M_2SO_4 \rightarrow M_2SeO_4 \rightarrow M_2TeO_4$ , and  $M_2CrO_4 \rightarrow M_2MoO_4 \rightarrow M_2WO_4$ , and also in going from  $M_2X^{(p)}O_4$  ( $X^{(p)} = S, Se, Te$ ) to  $M_2X^{(d)}O_4$  ( $X^{(d)} = Cr, Mo, W$ ) molecules are predicted with a fairly high accuracy, on a quantitative level.

(2) The internuclear distances  $R_g(A-B)$  determined by electron diffraction are apparent quantities. Namely,  $R_g(A-B)$  is the distance between nuclei A and B, averaged over all the vibrational states of the thermally equilibrium (i.e., described by the Boltzmann distribution) molecular ensemble. The apparent distance  $R_g(A-B)$  differs from the equilibrium internuclear distance  $R_e(A-B)$ . The correction  $\Delta R = R_g - R_e$  depends on temperature and on the structure and force field of the molecule. This correction can be different

for different pairs of nuclei. For “chemically bound” pairs of atoms, these corrections are usually positive. In the  $M_2XO_4$  molecules at temperatures of electron diffraction experiments ( $1100$ – $1800$  K) the corrections  $\Delta R = R_g - R_e$  to the internuclear distances X–O and M–O can be significant ( $\sim 0.02$ – $0.03$  Å, according to our estimates). In view of the facts that all the molecules  $M_2XO_4$  studied by gas-phase electron diffraction have similar structures and similar force fields and that the temperatures at which the electron diffraction patterns were taken (Table 6) differ not very strongly, the correction  $\Delta R = R_g - R_e$  can be expected to be similar for the pairs of nuclei of the same type in different  $M_2XO_4$  molecules. Hence, for structurally similar  $M_2XO_4$  molecules, comparison of trends in variation of the theoretical equilibrium internuclear distances  $R_e$  with the experimental data on apparent distances  $R_g$  is quite justified and can furnish useful information on the reliability of both experimental  $R_g$  values and theoretical predictions.

(3) In all the electron diffraction studies of  $M_2XO_4$  molecules [1–4], the diffraction patterns were interpreted assuming a structure with four symmetrically equivalent X–O distances and four symmetrically equivalent M–O distances ( $D_{2d}$  structure). Thus, in none of the electron diffraction studies [1–4] the possible coexistence of the  $D_{2d}$  isomer with the  $C_s$  isomer, predicted by the theory for all the  $M_2XO_4$  molecules except  $K_2SO_4$  (and probably also  $Rb_2SO_4$  and  $Cs_2SO_4$ ), was taken into account. At the same time, according to our estimates [14], under conditions of thermodynamic equilibrium the concentration of the  $bb'(C_s)$  isomer of the  $M_2WO_4$  molecules in the gas phase of rubidium and cesium tungstates can be comparable with that of the ground  $bb'(D_{2d})$  isomer: at temperatures of electron diffraction measurements [2], the relative content of the  $bb'(C_s)$  isomer should be about 30% for  $Rb_2WO_4$  ( $1070$  K) and 34% for

$\text{Cs}_2\text{WO}_4$  (1170 K). Significant concentrations of the excited isomers should also be expected for the other  $\text{M}_2\text{XO}_4$  molecules. For example, in potassium chromate vapor at the temperature of electron diffraction measurements [1] (1370 K) the content of the  $bb'(C_s)$  isomer, estimated from the thermodynamic functions that were calculated from the molecular constants found in this work, is as high as ~57%. Let us consider how the presence of the  $bb'(C_s)$  isomer of gaseous  $\text{M}_2\text{XO}_4$  molecules can disturb the geometries of these compounds evaluated assuming the  $bb(D_{2d})$  isomer as the sole species. Comparison of the equilibrium geometries of the  $bb$  and  $bb'$  isomers (Tables 1, 2) shows that the  $R_e(\text{X}-\text{O})_{bb}$  distance in the  $bb$  isomer and the average  $R_e(\text{X}-\text{O})$  distance in the  $bb'$  isomer [ $R_e^{\text{av}}(\text{X}-\text{O})_{bb'}$ ] are approximately equal:  $R_e^{\text{av}}(\text{X}-\text{O})_{bb'}$  exceeds  $R_e(\text{X}-\text{O})_{bb}$  by only 0.001–0.007 Å, with this difference tending to decrease in the series  $\text{Li}_2\text{XO}_4 \rightarrow \text{Na}_2\text{XO}_4 \rightarrow \text{K}_2\text{XO}_4$ . The average  $\text{O}\cdots\text{O}$  distances are also similar:  $R_e^{\text{av}}(\text{O}\cdots\text{O})_{bb'}$  differs from  $R_e^{\text{av}}(\text{O}\cdots\text{O})_{bb}$  in the  $\text{M}_2\text{XO}_4$  molecules, on the average, by 0.003 Å, with the largest difference (in the  $\text{Li}_2\text{TeO}_4$  molecule) being 0.015 Å. Hence, the structural parameters  $R_g(\text{X}-\text{O})$  and  $R_g(\text{O}\cdots\text{O})$  of the  $\text{XO}_4$  fragment of the  $\text{M}_2\text{XO}_4$  molecule, obtained when the electron scattering patterns of a mixture of the  $bb$  and  $bb'$  isomers are interpreted assuming existence of the  $bb$  isomer as a sole species, should not differ significantly from the actual structural parameters of the  $\text{XO}_4$  fragment in the  $bb$  isomer. As for the alkali metal–oxygen distances, in  $\text{Li}_2\text{XO}_4$  molecules the average distance  $R_e^{\text{av}}(\text{M}-\text{O})_{bb'}$  in the  $bb'$  isomer is somewhat shorter than the  $R_e(\text{M}-\text{O})_{bb}$  distance in the  $bb$  isomer, whereas in  $\text{Na}_2\text{XO}_4$  and  $\text{K}_2\text{XO}_4$  molecules it is slightly longer [for example, in the  $\text{K}_2\text{CrO}_4$  molecule the difference  $R_e^{\text{av}}(\text{M}-\text{O})_{bb'} - R_e(\text{M}-\text{O})_{bb}$  is 0.014 Å]. The trends in variation of the metal–oxygen distances in the  $bb$  and  $bb'$  isomers in the series  $\text{Li}_2\text{XO}_4 \rightarrow \text{Na}_2\text{XO}_4 \rightarrow \text{K}_2\text{XO}_4$  suggest that in  $\text{Rb}_2\text{XO}_4$  and  $\text{Cs}_2\text{XO}_4$  molecules the  $R_e^{\text{av}}(\text{M}-\text{O})_{bb'}$  distances should be longer than the  $R_e(\text{M}-\text{O})_{bb}$  distances by 0.02–0.03 Å. This fact suggests that for  $\text{M} = \text{K}, \text{Rb},$  and  $\text{Cs}$  the parameter  $R_g(\text{M}-\text{O})$  of the  $\text{M}_2\text{XO}_4$  molecules obtained from the electron diffraction data without taking isomerism into consideration will be somewhat overestimated. At the expected concentrations of the  $bb'$  isomer the overestimation can reach 0.01–0.02 Å.

Note that the errors in estimating the apparent (obtained without considering isomerism) root-mean-square vibration amplitudes of the nuclei in  $\text{M}_2\text{XO}_4$  molecules should be considerably more significant. These errors originate from line broadening in the radial distribution curves  $f(R)$  for the total diffraction pattern as compared to the curve  $f(R)_{bb}$  corresponding

to electron scattering on the  $bb$  molecules only. The broadening is due to summation of two functions:  $f(R)_{bb}$  from the  $bb$  isomer and  $f(R)_{bb'}$  from the  $bb'$  isomer. Furthermore, the structural peaks in the  $f(R)_{bb'}$  curve should consist of several components. For example, in the radial distribution curve  $f(R)_{bb'}$  the peak corresponding to the  $\text{M}-\text{O}$  distance will consist of two peaks ( $\text{M}-\text{O}$  and  $\text{M}-\text{O}'$ ), and the peak corresponding to the  $\text{X}-\text{O}$  distance, of three peaks ( $\text{X}-\text{O}$ ,  $\text{X}-\text{O}'$ , and  $\text{X}-\text{O}''$ ) (Fig. 1; Tables 1, 2). In what follows we discuss only the internuclear distances obtained in electron diffraction experiments (the experimental and theoretical mean vibration amplitudes in alkali metal tungstates are compared in our previous paper [14]).

On the whole, the above arguments show that comparison of the internuclear distances in  $\text{M}_2\text{XO}_4$  molecules, determined experimentally by electron diffraction [1–4] and calculated *ab initio* for the ground isomer  $bb(D_{2d})$ , is justified. However, additional discussion is required for the  $\text{Rb}_2\text{SO}_4$  and  $\text{Cs}_2\text{SO}_4$  molecules, because the results of our calculations do not rule out the existence of the bistridentate ( $tt$ ,  $C_{2v}$  symmetry) isomer of these molecules, close to the  $bb(D_{2d})$  isomer in energy. Comparison of the internuclear distances in the  $bb(D_{2d})$  and  $tt(C_{2v})$  structures (Tables 1, 2) shows that the average  $\text{X}-\text{O}$  and  $\text{O}\cdots\text{O}$  distances in both structures differ insignificantly. At the same time, the metal–oxygen distances in the  $tt(C_{2v})$  isomer are considerably longer than those in the  $bb(D_{2d})$  isomer: the difference  $R_e^{\text{av}}(\text{K}-\text{O})_{tt} - R_e(\text{K}-\text{O})_{bb}$  in the  $\text{K}_2\text{SO}_4$  molecule is as large as 0.175 Å. For  $\text{Rb}_2\text{SO}_4$  and  $\text{Cs}_2\text{SO}_4$ , the difference between the metal–oxygen distances in the  $tt(C_{2v})$  and  $bb(D_{2d})$  structures should be similar or even greater. Hence, if the content of the  $tt(C_{2v})$  isomer in the vapors of rubidium and cesium sulfates is significant, the apparent metal–oxygen distance determined by gas-phase electron diffraction without taking into account the isomerism of the  $\text{Rb}_2\text{SO}_4$  and  $\text{Cs}_2\text{SO}_4$  molecules should appreciably exceed the distance expected for the  $bb(D_{2d})$  isomer.

Let us start comparison of the calculation results and electron diffraction data with structural parameters of the  $\text{XO}_4$  fragment. Our calculations show that in the series  $\text{Li}_2\text{XO}_4 \rightarrow \text{Na}_2\text{XO}_4 \rightarrow \text{K}_2\text{XO}_4$  the  $R_e(\text{X}-\text{O})$  distance is approximately constant, which allows us to expect that in going to the rubidium ( $\text{Rb}_2\text{XO}_4$ ) and cesium ( $\text{Cs}_2\text{XO}_4$ ) salts it will not appreciably change either. This conclusion is consistent with the electron diffraction data (Table 6): The  $R_g(\text{X}-\text{O})$  distances (within determination error) are the same in molecules with different  $\text{M}$ . Variation of the theoretical  $\text{X}-\text{O}$  distances  $R_e(\text{X}-\text{O})$  in the series  $\text{M}_2\text{CrO}_4 \rightarrow \text{M}_2\text{MoO}_4 \rightarrow \text{M}_2\text{WO}_4$  agrees with experimentally observed variation

of  $R_g(X-O)$ :  $R_e(Mo-O) - R_e(Cr-O) = 0.15$  Å for  $K_2XO_4$  (Table 2),  $R_g(Mo-O) - R_g(Cr-O) = 0.12 \pm 0.03$  Å for  $Cs_2XO_4$  (Table 6);  $R_e(W-O) - R_e(Mo-O) = 0.01$  Å for  $K_2XO_4$  (Table 2),  $R_g(W-O) - R_g(Mo-O) = 0.02 \pm 0.02$  Å for  $Rb_2XO_4$  and  $Cs_2XO_4$  (Table 6). The experimental parameters  $R_g(X-O)$  of the  $M_2CrO_4$ ,  $M_2MoO_4$ , and  $M_2WO_4$  molecules exceed the theoretical  $R_e(X-O)$  values by 0.06–0.07 (X = Cr) or 0.03–0.05 Å (X = Mo, W). This discrepancy is not very large, taking into account the expected corrections  $\Delta R(X-O) = R_g(X-O) - R_e(X-O)$  and the errors of  $R_g^{exp}$  and  $R_e^{theor}$ . At the same time, for alkali metal sulfate molecules comparison of the theoretical ( $R_e$ ) and experimental ( $R_g$ ) S–O distances, taking into account the similar corrections and possible errors, shows that the theoretical data are not quite consistent with the experiment: the experimental  $R_g(S-O)$  values are lower by 0.03–0.04 Å than those predicted by the theory. The theoretical bond angles  $\alpha_e(OXO)$  in the  $M_2XO_4$  molecules of the  $D_{2d}$  symmetry agree with the experimental  $\alpha_g(OXO)$  values.

Let us now compare the metal–oxygen distances. Direct comparison can be performed for two molecules only:  $K_2SO_4$  and  $K_2CrO_4$ . The  $R_g(K-O)$  distances in the potassium sulfate and chromate molecules, according to [1], are equal within the determination error ( $\pm 0.03$  Å). Our calculations give a similar result: In the  $K_2SO_4$  and  $K_2CrO_4$  molecules of the  $D_{2d}$  symmetry,  $R_e(K-O)$  differs by only 0.015 Å (Tables 1, 2). At the same time, the absolute values of the K–O distances calculated by us and found in [1] differ considerably: The measured  $R_g(K-O)$  values are 0.07–0.09 Å lower than the theoretical  $R_e(K-O)$  values. After taking into account the correction  $\Delta R = R_g - R_e$ , this discrepancy becomes even larger, reaching 0.10–0.12 Å, which considerably exceeds the total error in  $R_g$  and  $R_e$ . Thus, our calculations suggest that the parameters  $R_g(K-O)$  obtained in [1] for  $K_2SO_4$  and  $K_2CrO_4$  are erroneous.

The metal–oxygen distances in the molecules of rubidium and cesium salts  $M_2XO_4$  determined by gas-phase electron diffraction can be compared to the calculated M–O distances in the corresponding molecules with lighter alkali metal using the above-mentioned system of ionic radii from [21]. The  $R_e(M-O)$  distance in the  $Rb_2XO_4$  and  $Cs_2XO_4$  can be estimated by the formula:

$$R_e(M-O) = R_e(K-O) + R(M^+) - R(K^+), \quad (2)$$

where M = Rb or Cs,  $R_e(K-O)$  is the K–O distance in the corresponding  $K_2XO_4$  molecule calculated by us, and  $R(M^+)$  and  $R(K^+)$  are the ionic radii from [21].

The  $R_e(Rb-O)$  distances in the ground ( $D_{2d}$ ) iso-

mers of the  $Rb_2MoO_4$  and  $Rb_2WO_4$  molecules, calculated by this procedure, are 2.70 and 2.71 Å, respectively. The former value agrees with the distance  $R_g(Rb-O) = 2.74 \pm 0.02$  Å in the  $Rb_2MoO_4$  molecule, measured in [2]. However, the value of  $2.69 \pm 0.02$  Å obtained in the same study [2] for  $R_e(Rb-O)$  in  $Rb_2WO_4$  is probably erroneous: it is appreciably lower than  $R_g(Rb-O)$  expected by us ( $\sim 2.75$  Å) and contradicts the trend, revealed by *ab initio* calculations, toward an increase in the alkali metal–oxygen distance in going from molybdates to tungstates.

Calculation by formula (2) gives the following M–O internuclear distance in the  $D_{2d}$  isomers of  $Cs_2XO_4$  molecules:  $R_e(Cs-O)$  2.76 Å in  $Cs_2SO_4$ , 2.77 Å in  $Cs_2CrO_4$ , 2.82 Å in  $Cs_2MoO_4$ , and 2.83 Å in  $Cs_2WO_4$ . The value of  $R_e(Cs-O)$  estimated for  $Cs_2SO_4$  is quite consistent with  $R_g(Cs-O) = 2.80 \pm 0.05$  Å, measured for this molecule by electron diffraction [3], and both the result of [3] and calculated value contradict the  $R_g(Cs-O)$  distance found for  $Cs_2SO_4$  in [1]:  $2.60 \pm 0.03$  Å. This fact supports our conclusion that the metal–oxygen distances in  $M_2XO_4$  molecules determined by electron diffraction in [1] are erroneous. Furthermore, agreement between the theoretically predicted Cs–O distance in the  $D_{2d}$  isomer of the  $Cs_2SO_4$  molecule and the experimental value [3] shows that the  $D_{2d}$  isomer is, indeed, the ground isomer of  $Cs_2SO_4$  and that no significant amounts of the *tt*( $C_{2v}$ ) isomer are present. The same conclusion is apparently valid for  $Rb_2SO_4$ .

The  $R_e(Cs-O)$  distance evaluated by us for the *bb*( $D_{2d}$ ) isomer of the  $Cs_2CrO_4$  molecule is 0.08 Å shorter than the apparent experimental [4]  $R_g(Cs-O)$  distance,  $2.85 \pm 0.04$  Å. The difference  $R_g^{exp} - R_e^{theor} = 0.08$  Å exceeds the expected correction for this pair of nuclei  $\Delta R = R_g - R_e$ . Such a difference between the observed apparent (obtained without considering isomerism) Cs–O distance in the  $Cs_2CrO_4$  molecule and the  $R_e(Cs-O)$  distance theoretically predicted for the *bb*( $D_{2d}$ ) isomer of this molecule is an indirect evidence of an appreciable content of the *bb'*( $C_s$ ) isomer of  $Cs_2CrO_4$  in the phase of cesium chromate. However, the deviation of the theoretical prediction from the experiment can also be due to significant errors in determination of  $R_g^{exp}$  and calculation of  $R_e^{theor}$ .

In the light of our results, the  $R_g(Cs-O)$  in the  $Cs_2MoO_4$  and  $Cs_2WO_4$  molecules measured in [2] (Table 6) seem to be appreciably underestimated (by 0.05–0.10 Å); the same is true for the  $R_g(Rb-O)$  distance in the  $Rb_2WO_4$  molecule, measured in [2].

To conclude: Unfortunately, available structural data obtained for  $M_2XO_4$  molecules by gas-phase electron diffraction are often insufficiently reliable.

This is primarily due to problems with interpretation of high-temperature electron diffraction patterns of nonrigid molecules. Large vibration amplitudes smear the diffraction patterns, and the structural information related to distances between the alkali metal atoms and atoms of the  $\text{XO}_4$  fragment is "washed out." Another complicating factor is the structural isomerism of the  $\text{M}_2\text{XO}_4$  molecules, which was taken into account in none of the published electron diffraction studies.

We believe that it would be useful to perform additional diffraction and spectroscopic studies of  $\text{M}_2\text{XO}_4$  molecules and interpret the results taking into account the structural features of these molecules revealed in this work.

### ACKNOWLEDGMENTS

The work was supported by the Ministry of Education of the Russian Federation (Competitive Center for Basic Research, St. Petersburg State University), project no. 97-0-9.1-262.

### REFERENCES

1. Spiridonov, V.P. and Lutoshkin, B.N., *Vestn. Mosk. Gos. Univ., Ser. 2: Khim.*, 1970, vol. 11, no. 5, pp. 509–512.
2. Ezhov, Yu.S. and Sarvin, A.P., *Zh. Strukt. Khim.*, 1980, vol. 21, no. 5, pp. 40–45.
3. Kulikov, V.A., Ugarov, V.V., and Rambidi, N.G., *Zh. Strukt. Khim.*, 1982, vol. 23, no. 1, pp. 184–187.
4. Girichev, G.V., Giricheva, N.I., Kuligin, E.A., and Krasnov, K.S., *Zh. Strukt. Khim.*, 1983, vol. 24, no. 1, pp. 63–69.
5. Dvorkin, M.I., *Cand. Sci. (Phys.-Math.) Dissertation*, Leningrad, 1977.
6. Spoliti, M., in *Matrix Isolation Spectroscopy. Lectures and Discussions (Montpellier, 1980)*, Dordrecht: NATO Adv. Study Inst., 1981, pp. 473–484.
7. Beattie, I.R., Ogden, J.S., and Price, D.D., *J. Chem. Soc., Dalton Trans.*, 1982, no. 3, pp. 505–510.
8. Bencivenni, L. and Gingerich, K.A., *J. Chem. Phys.*, 1982, vol. 76, no. 1, pp. 53–56.
9. Nagarathna, H.M., Bencivenni, L., and Gingerich, K.A., *J. Chem. Phys.*, 1984, vol. 81, no. 2, pp. 591–598.
10. Brisdon, A.K., Gomme, R.A., and Ogden, J.S., *J. Phys. Chem.*, 1991, vol. 95, no. 7, pp. 2927–2931.
11. Ramondo, F., Bencivenni, L., Caminiti, R., and Sadun, C., *Chem. Phys.*, 1991, vol. 151, no. 2, pp. 179–186.
12. Sliznev, V.V. and Solomonik, V.G., *Koord. Khim.*, 1996, vol. 22, no. 9, pp. 699–705.
13. Marenich, A.V. and Solomonik, V.G., *Zh. Neorg. Khim.*, 2001, vol. 46, no. 3, pp. 462–466.
14. Solomonik, V.G. and Marenich, A.V., *Zh. Fiz. Khim.*, 2000, vol. 74, no. 1, pp. 94–102.
15. Schmidt, M.W., Baldrige, K.K., Boatz, J.A., Elbert, S.T., Gordon, M.S., Jensen, J.H., Koseki, S., Matsunaga, N., Nguyen, K.A., Su, S.J., Windus, T.L., Dupuis, M., and Montgomery, J.A., *J. Comput. Chem.*, 1993, vol. 14, no. 11, pp. 1347–1363.
16. Stevens, W.J., Basch, H., and Krauss, M., *J. Chem. Phys.*, 1984, vol. 81, no. 12, pp. 6026–6033.
17. Stevens, W.J., Krauss, M., Basch, H., and Jasien, P.G., *Can. J. Chem.*, 1992, vol. 70, no. 2, pp. 612–630.
18. Huzinaga, S., Andzelm, J., Klobukowski, M., Radzio-Andzelm, E., Sakai, J., and Tatewaki, H., *Gaussian Basis Sets for Molecular Calculations (Phys. Sci. Data, vol. 16)*, Amsterdam: Elsevier, 1984, vol. 8.
19. Davidson, E.R. and Silver, D.W., *Chem. Phys. Lett.*, 1977, vol. 52, no. 3, pp. 403–406.
20. Solomonik, V.G., *Doctoral (Chem.) Dissertation*, Moscow, 1993.
21. Dunbar, I.H., MacLagan, R.G.A.R., and Parr, R.G., *J. Mol. Struct.*, 1974, vol. 23, no. 1, pp. 121–129.
22. Choudary, U.V., Gingerich, K.A., and Kingcade, J.E., *J. Less-Common Met.*, 1975, vol. 42, no. 2, pp. 111–126.
23. Kuligina, L.A. and Semenov, G.A., *Vestn. Leningr. Gos. Univ., Ser. Fiz., Khim.*, 1985, no. 18, pp. 39–45.
24. Lopatin, S.I., Semenov, G.A., Kirsanov, D.O., and Shugurov, S.M., *Zh. Obshch. Khim.*, 2000, vol. 70, no. 3, pp. 383–385.